The first Inverse-Scattering-Series internal multiple *elimination* method for a *multi*-dimensional subsurface

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SUMMARY

The ISS (Inverse-Scattering-Series) internal-multiple attenuation algorithm (Araújo et al. (1994),Weglein et al. (1997) and Weglein et al. (2003)) is the most effective algorithm today for internal multiple removal. It is the only multi-dimensional method that can predict the correct time and approximate amplitude for all internal multiples at once, without any subsurface information. When combined with an energy minimization adaptive subtraction, the ISS internal-multiple attenuation algorithm can effectively eliminate internal multiples when the primaries and internal multiples are separated. However, under many offshore and onshore circumstances where internal multiples are often proximal to or interfering with primaries, the criteria of energy minimization adaptive subtraction can fail (e.g., the energy can increase when a multiple is removed from a destructively interfering primary and multiple). Therefore, Weglein (2014) proposed a three-pronged strategy for providing an effective response to removing internal multiples without damaging interfering primaries. Currently, there is no capability available in the petroleum industry that addresses that type of serious and frequently occurring challenge. A major component of the strategy is to develop an internal-multiple elimination algorithm that can predict both the correct amplitude and correct time for all internal multiples. The initial idea to achieve an elimination algorithm is developed by Weglein and Matson (1998) by removing attenuation factors (the difference between the predicted internal multiples and true internal multiples) using reflection data. There are early discussions in Ramírez (2007). Based on the ISS attenuation algorithm and the initial idea for elimination, Herrera and Weglein (2012) formulated an ISS algorithm for a normal incident wave on a 1D earth, that eliminate first-order internal multiples generated by the shallowest reflector and further attenuates first-order internal multiples from deeper reflectors. Zou and Weglein (2014) then advanced and extended these initial contributions for the pre-stack and for all first order internal multiples generated at all reflectors. In this paper, we further extend the 1-D elimination algorithm and provide the first ISS multi-dimensional elimination method for all first order internal multiples.

INTRODUCTION

The ISS (Inverse-Scattering-Series) allows all seismic processing objectives, e.g., free-surface-multiple removal and internal-multiple removal to be achieved directly in terms of data, without any need for or estimation of the earth's properties. The ISS internal-multiple attenuation algorithm is the only method today that can predict the correct time and approximate and well-understood amplitude for all first-order internal multiples generated from all reflectors, at once, without any subsurface information. If the multiple to be removed is isolated from other events, then the energy minimization adaptive subtraction can fill the gap between the attenuation algorithm amplitude prediction and the internal multiples plus, e.g., all preprocessing factors that are outside the assumed physics of the subsurface and acquisition. However primary and multiple events can often interfere with each other in both on-shore and off-shore seismic data. In these cases, the criteria of energy minimization adaptive subtraction may fail and completely removing internal multiples becomes more challenging and beyond the current capability of the petroleum industry.

For dealing with this challenging problem, Weglein (2014) proposed a three-pronged strategy including

1. Develop the ISS prerequisites for predicting the reference wave field and to produce de-ghosted data.
2. Develop internal-multiple elimination algorithms from ISS.
3. Develop a replacement for the energy-minimization criteria for adaptive subtraction.

To achieve the second part of the strategy, that is, to upgrade the ISS internal-multiple attenuation algorithm to elimination algorithm, the strengths and limitations of the ISS internal-multiple attenuation algorithm are noted and reviewed. The ISS internal-multiple attenuation algorithm always attenuates all internal multiples from all reflectors at once, automatically and without any subsurface information. That unique strength always present and is independent of the circumstances and complexity of the geology and the play. However, there are two well-understood limitations of this ISS internal-multiple attenuation algorithm

1. It may generate spurious events due to internal multiples treated as sub-events.
2. It is an attenuation algorithm not an elimination algorithm.

The first item is a shortcoming of the leading order term (the term used to derive the current attenuation algorithm), when taken in isolation, **but is not an issue for the entire ISS internal-multiple removal capability**. It is anticipated by the ISS and higher order ISS internal multiple terms exist to precisely remove that issue of spurious events prediction. When taken together with the higher order terms, the ISS internal multiple removal algorithm no longer experiences spurious events prediction. Ma et al. (2012) , H. Liang and Weglein (2012) and Ma and Weglein (2014) provided those higher order terms for spurious events removal.

In a similar way, there are higher order ISS internal multiple terms that provide the elimination of internal multiples when taken together with the leading order attenuation term. The initial idea is provided by Weglein and Matson (1998) in which the attenuation factor, which is a collection of extra transmission coefficients and is the difference between attenuation and elimination, is systematically studied. Later there are furthur discussions in Ramírez (2007). Several extensions are proposed based on the initial idea. Herrera and Weglein (2012) proposed an algorithm for internal multiple elimination for all first order internal multiples generated at the first reflector. Benefited from the previous work, Zou and Weglein (2014) proposed an new algorithm that can eliminate all first order internal multiples for all reflectors for a 1D earth. In this paper, we further extend the previous elimination algorithm and provide the first ISS multi-dimensional elimination method for all first order internal multiples. The new elimination algorithm retains the benefits of the attenuation algorithm, including not requiring any subsurface information and unlike stripping methods, removes all first-order internal multiples from all subsurface reflectors at once.

THE ISS INTERNAL-MULTIPLE ATTENUATION ALGORITHM AND THE INITIAL IDEA FOR INTERNAL MULTIPLE ELIMINATION

The ISS internal-multiple attenuation algorithm is first given by Araújo et al. (1994) and Weglein et al. (1997). The 1D normal incidence version of the algorithm is presented as follows (The 2D version is given in Araújo et al. (1994),Weglein et al. (1997) and Weglein et al. (2003) and the 3D version is a straightforward extension.),

$$b_3(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z''). \tag{1}$$

Where $b_1(z)$ is the constant velocity Stolt migration of the data of a 1D normal incidence spike plane wave. ε_1 and ε_2 are two small positive numbers introduced to avoid self interactions. $b_3^{IM}(k)$ is the predicted internal multiples in the vertical wavenumber domain. This algorithm can predict the correct time and approximate amplitude of all first-order internal multiples at once without any subsurface information.

The procedure of predicting a first-order internal multiple generated at the shallowest reflector is shown in figure 1. The ISS internal-multiple attenuation algorithm automatically uses three primaries in the data to predict a first-order internal multiple. (Note that this algorithm is model type independent and it takes account all possible combinations of primaries that can predict internal multiples.) From this figure we can see, every sub event on the left hand side experiences several phenomena making its way down to the earth then back to the receiver. When compared with the internal multiple on the right hand side, the events on the left hand side have extra transmission coefficients as shown in red. Multiplying all those extra transmission coefficients, we get the AF (attenuation factor) - $T_{01}T_{10}$ for this first-order internal multiple generated at the shallowest reflector. And all first-order internal multiples generated at the shallowest reflector have the same attenuation factor.

Figure 2 shows the procedure of predicting a first-order internal multiple generated at the next shallowest reflector. In this example, the attenuation factor is $(T_{01}T_{10})^2(T_{12}T_{21})$.

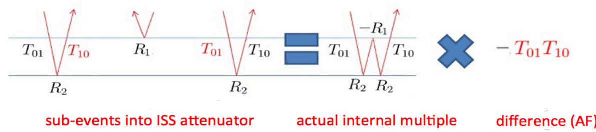

Figure 1: an example of the attenuation factor of a first-order internal multiple generated at the shallowest reflector, notice that all red terms are extra transmission coefficients

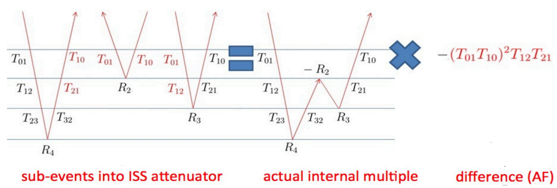

Figure 2: an example of the attenuation factor of a first-order internal multiple generated at the next shallowest reflector, notice that all red terms are extra transmission coefficients

The attenuation factor for predicting a multiple generated by the i^{th} reflector, AF_j, is given by the following:

$$AF_j = \begin{cases} T_{0,1}T_{1,0} \quad (for\ j=1) \\ \prod_{i=1}^{j-1}(T_{i-1,i}^2 T_{i,i-1}^2)T_{j,j-1}T_{j-1,j} \quad (for\ 1<j<J) \end{cases} \tag{2}$$

The subscript j represents the generating reflector, and J is the total number of interfaces in the model. The interfaces are numbered starting with the shallowest location. The attenuation factor is a collection of extra transmission coefficients and is the difference between attenuation and elimination. Weglein and Matson (1998) studied the attenuation factor and provide the initial idea and algorithm to remove the attenuation factor by reflection data to achieve the elimination.

As discussed in Weglein and Matson (1998), the attenuation algorithm prediction contains the attenuation factor and in order to develop an elimination algorithm, we should remove the attenuation factor. However, the attenuation factor is expressed using transmission coefficients. Since the data contains reflection coefficients, the idea is to use reflection coefficients to represent the transmission coefficient such that we can remove the attenuation factor by the data (which contains reflection coefficients) without any subsurface information. For example, to remove AF_1 in the prediction, we have

$$\frac{1}{AF_1} = \frac{1}{T_{0,1}T_{1,0}} = \frac{1}{1-R_1^2} = 1 + R_1^2 + R_1^4 + \ldots \tag{3}$$

where the first term 1 corresponds to the attenuation algorithm, the term R_1^2 corresponds to the first higher order term towards elimination and so on. A detailed discussion can be found in Weglein and Matson (1998) and Ramírez (2007).

PREVIOUS WORK: WILBERTH'S AND YANGLEI'S EXTENSIONS

Based on the ISS internal-multiple attenuation algorithm and the initial idea for elimination, Herrera and Weglein (2012) formulated an algorithm for internal multiple elimination for all first order internal multiples generated at the first reflector. The first term in this elimination algorithm is the current attenuator (equation (1)), which corresponds to the first term 1 in equation (3). The second term in this elimination algorithm is shown as follows (The complete algorithm is given in Herrera and Weglein (2012)),

$$\int_{-\infty}^{+\infty} dz b_1(z)e^{2ikz} \int_{-\infty}^{z-\varepsilon} dz' F(z')e^{-2ikz'} \int_{z'+\varepsilon}^{+\infty} dz'' b_1(z'')e^{2ikz''} \tag{4}$$

Where $F(z)$ is

$$F(z) = \int_{-\infty}^{+\infty} d(2k)e^{-2ikz} \int_{-\infty}^{+\infty} dz' b_1(z')e^{2ikz'} \times \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' b_1(z'')e^{-2ikz''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' b_1(z''')e^{2ikz'''} \tag{5}$$

The purpose of $F(z)$ in the middle integrand is to provide R^2 term shown in equation (3) in order to remove the attenuation factor.

Benefited from the initial idea and Wilberth's work, Zou and Weglein (2014) formulated an elimination algorithm that can eliminate all first order internal multiples for all reflectors for a 1D earth. Below shows the elimination algorithm, where $b_1(k,z)$ is the water speed uncollapsed Stolt migration of the data; $b_E(k,2q)$ is the elimination algorithm prediction in wavenumber domain; $F[b_1(k,z)]$ and $g(k,z)$ are two intermediate functions.

$$b_E(k,2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k,z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k,z')] \times \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k,z'') \tag{6}$$

$$F[b_1(k,z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{[1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g(k,z''') e^{-iq'z'''}]^2} \times \frac{1}{1 - |\int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k,z'') e^{iq'z''}|^2} \tag{7}$$

$$g(k,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g(k,z''') e^{-iq'z'''}} \tag{8}$$

All the ISS internal multiple removal algorithms predict internal multiples at once, without requiring any subsurface information. For first -order internal-multiples generated at the shallowest reflector, both extensions are able to eliminate. For first -order internal-multiples generated at the deeper reflectors, the Wilberth's extension is still an attenuator and is expected to predict better amplitude than the current attenuator while the Yanglei's extension is an eliminator.

THE FIRST INVERSE-SCATTERING-SERIES INTERNAL MULTIPLE ELIMINATION METHOD FOR A MULTI-DIMENSIONAL SUBSURFACE

The Inverse-Scattering-Series contains an internal-multiple elimination sub-series. Since the internal multiple attenuation algorithm is capable to predict the correct time and approximate amplitude for all internal multiples, if we can isolate all terms that can predict the same time as the attenuation algorithm by the initial elimination idea (removing the attenuation factor by the reflection data) in the Inverse-Scattering-Series, then adding all these terms together will give us an elimination algorithm. Since the Inverse-Scattering-Series is a multi-D series, the elimination algorithm/terms identified is a multi-D algorithm. Benefited from the previous work, we propose a Inverse-Scattering-Series internal multiple elimination method that can eliminate all first-order internal multiples for all reflectors for a multi–dimensional subsurface. Below shows a 2D version of a higher order term in the elimination algorithm.

$$b_E(k_s,k_g,q_g+q_s) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dk_1 dk_2 \int_{-\infty}^{+\infty} dz_1 b_1(k_g,k_1,z_1) e^{i(q_g+q_1)z_1} \times \int_{-\infty}^{z_1-\varepsilon} dz_2 F(k_1,k_2,z_2) e^{-i(q_1+q_2)z_2} \int_{z_2+\varepsilon}^{+\infty} dz_3 b_1(k_2,k_s,z_3) e^{i(q_2+q_s)z_3} \tag{9}$$

$$F(k_1,k_2,z) = \int_{-\infty}^{+\infty} d(q_1+q_2) e^{-i(q_1+q_2)z} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dk' dk'' \int_{-\infty}^{+\infty} dz' b_1(k_1,k',z') e^{i(q_1+q')z'} \times \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k',k'',z'') e^{-i(q'+q'')z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g(k'',k_2,z''') e^{i(q''+q_2)z'''} \tag{10}$$

$$g(k_1,k_2,z) = \int_{-\infty}^{+\infty} d(q_1+q_2)e^{-i(q_1+q_2)z} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dk'dk'' \int_{-\infty}^{+\infty} dz' b_1(k_1,k',z')e^{i(q_1+q')z'}$$
$$\times \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' b_1(k',k'',z'')e^{-i(q'+q'')z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g(k'',k_2,z''')e^{i(q''+q_2)z'''} \quad (11)$$

Similar to the extension in the previous work in Zou and Weglein (2014) , $F(k_1,k_2,z)$ and $g(k_1,k_2,z)$ are two intermediate functions. Substituting into equation (9) provides a higher order term in the elimination sub-series.

Combining all of these kind of higher order terms provides the elimination algorithm in a 2D earth. The elimination algorithm for a 3D earth is a straightforward extension.The complete subseries is given in Zou et al. (2016).

CONCLUSION

The ISS internal multiple elimination algorithm is a part of the three-pronged strategy which is a direct response to current seismic processing and interpretation challenge when primaries and internal multiples are proximal to and/or interfere with each other in either on-shore and off-shore plays. This paper extends and generalizes the earlier 1D ISS internal multiple elimination method to provide an algorithm that eliminates all first-order internal multiples in a multi-D earth. This elimination algorithm retains the stand-alone benefits of the ISS internal multiple attenuation algorithm that can predict all internal multiples at once (in contrast to stripping methods that remove multiples layer by layer and require subsurface information) and requiring no subsurface information.

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